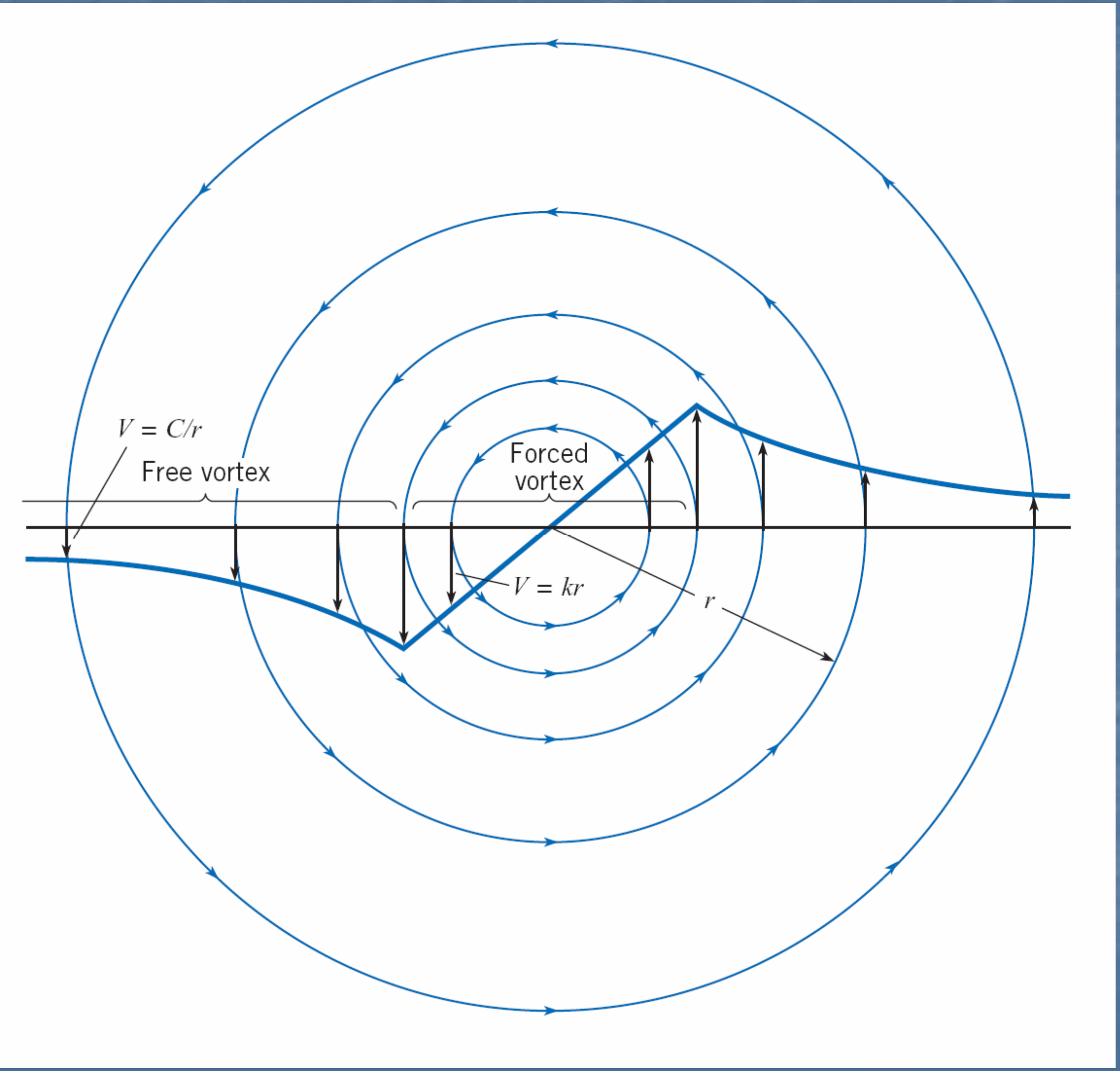
Pressure Distribution in Rotational flows

Combination of free and forced vortex



Apply Euler Equation, we have,

$$-\frac{d}{dr}(p+\gamma z)=\rho a_r$$

Acceleration the radial direction is given as

$$a_r=-\frac{V^2}{r}$$

For liquid rotating as a rigid body, i.e. $V = \omega r$

$$\frac{d}{dr}\left(p + \gamma z\right) = \rho r \omega^2$$

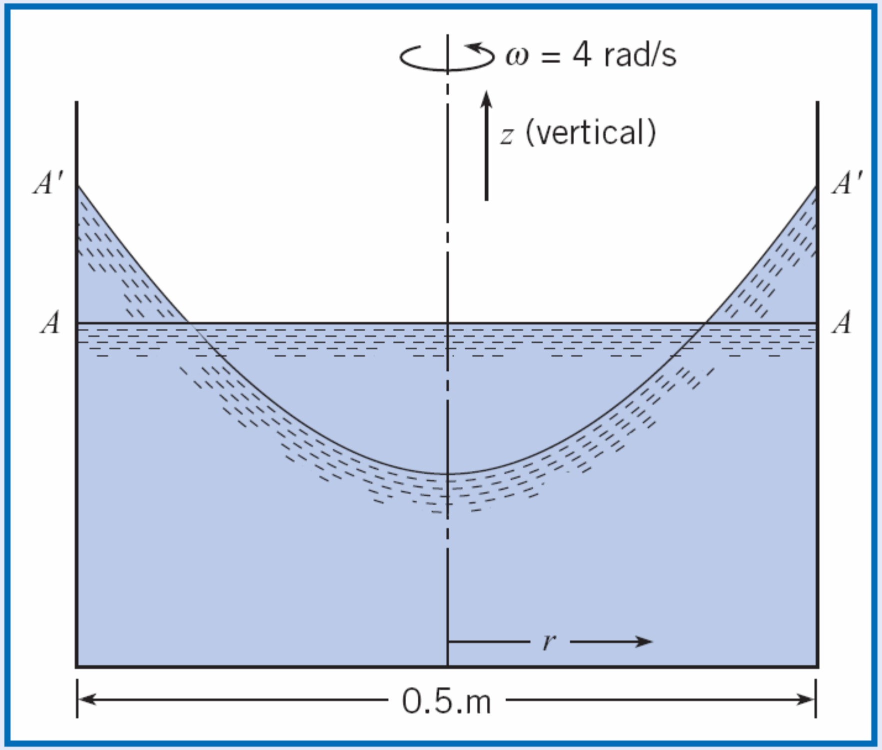
Integrating the above equation

$$\left(p + \gamma z\right) = \frac{\rho r^2 \omega^2}{2} + C$$

$$\left(\frac{p}{\gamma} + z - \frac{\omega^2 r^2}{2g}\right) = C$$

<u>The above equation describes the pressure variation in rotating flow</u>

Find the elevation difference between the liquid at the center and the wall during rotation?



Applying $\left(\frac{p}{\gamma} + z - \frac{\omega^2 r^2}{2g}\right) = C$ of the liquid surface

at the center will reduces to

$$\left(z - \frac{\omega^2 r^2}{2g}\right) = C$$

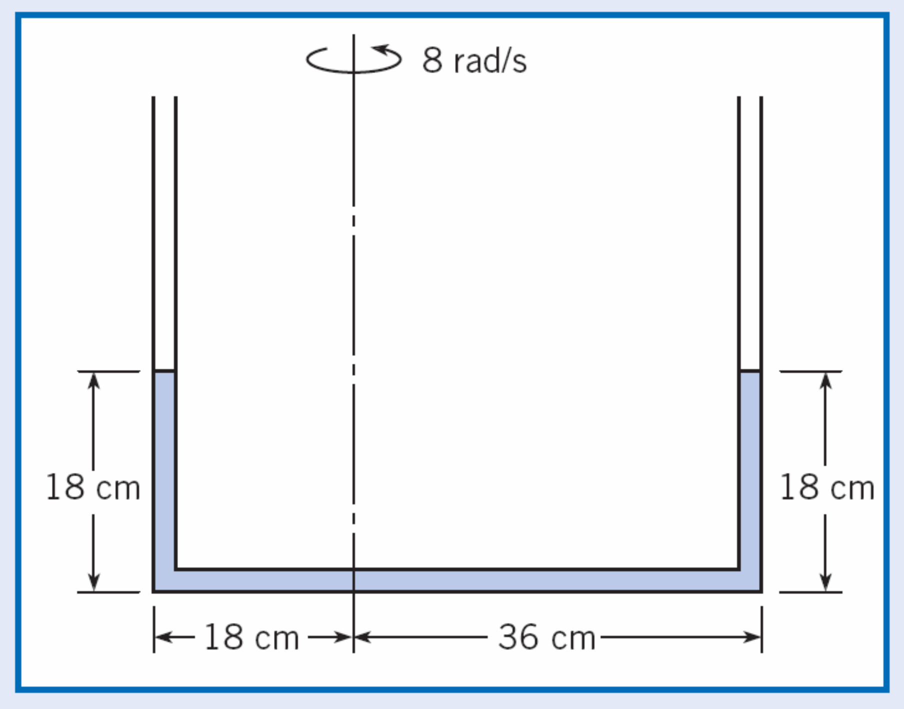
$r = 0$ $\quad C = z_0$

$$z = \left(z_0 + \frac{\omega^2 r^2}{2g}\right)$$

at $(r = \frac{d}{2})$ $\quad (z)_{d/2} = \left(z_0 + \frac{\omega^2 (d/2)^2}{2g}\right)$ $\quad (z)_{d/2} - z_0 = \left(\frac{\omega^2 (d/2)^2}{2g}\right)$

Find the new level of water during rotation?

Applying $\left(\frac{p}{\gamma} + z - \frac{\omega^2 r^2}{2g}\right) = C$ about the line of rotation



P=0 as atmospheric, then

$$\gamma z_l - \left(\frac{\rho(r_l\omega)^2}{2}\right) = \gamma z_r - \left(\frac{\rho(r_r\omega)^2}{2}\right)$$

$$z_l - z_r = \left(\frac{\omega^2}{2g}\right)\left(r_l^{\,2} - r_r^{\,2}\right)$$

$$z_l + z_r = 0.36$$

$$z_l = 2.1cm \quad z_r = 33.9\,cm$$

Bernoulli's Equation in Irrotational flow

$$\Omega_z = \frac{1}{2}\left(\frac{dV}{dr} + \frac{V}{r}\right)$$

For Irrotational Flow, $\left(\frac{dV}{dr} = -\frac{V}{r}\right)$

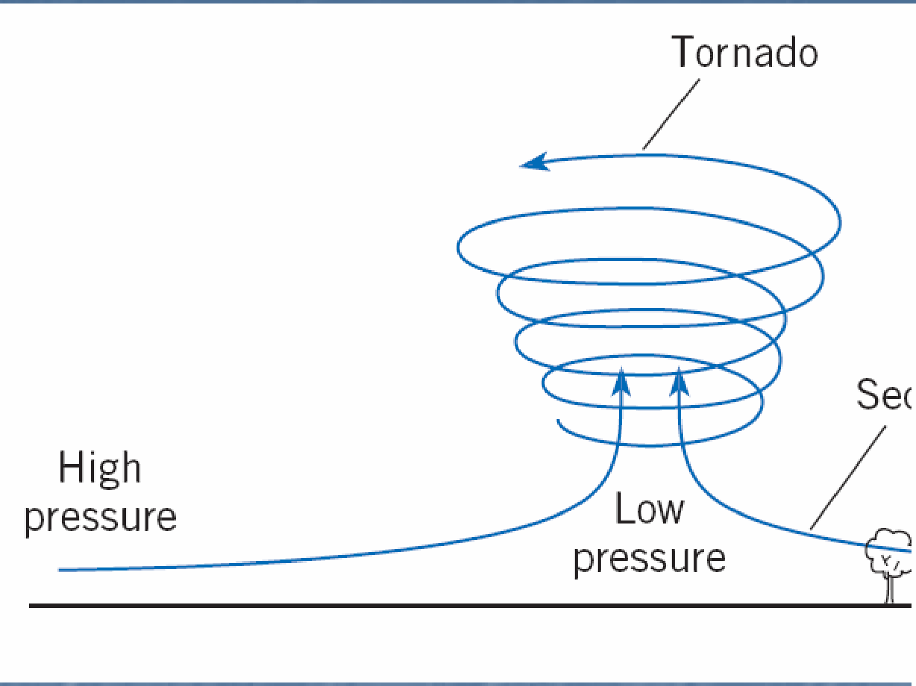
Apply Euler Equation, we have, $-\frac{d}{dr}(p + \gamma z) = -\rho\frac{V^2}{r}$

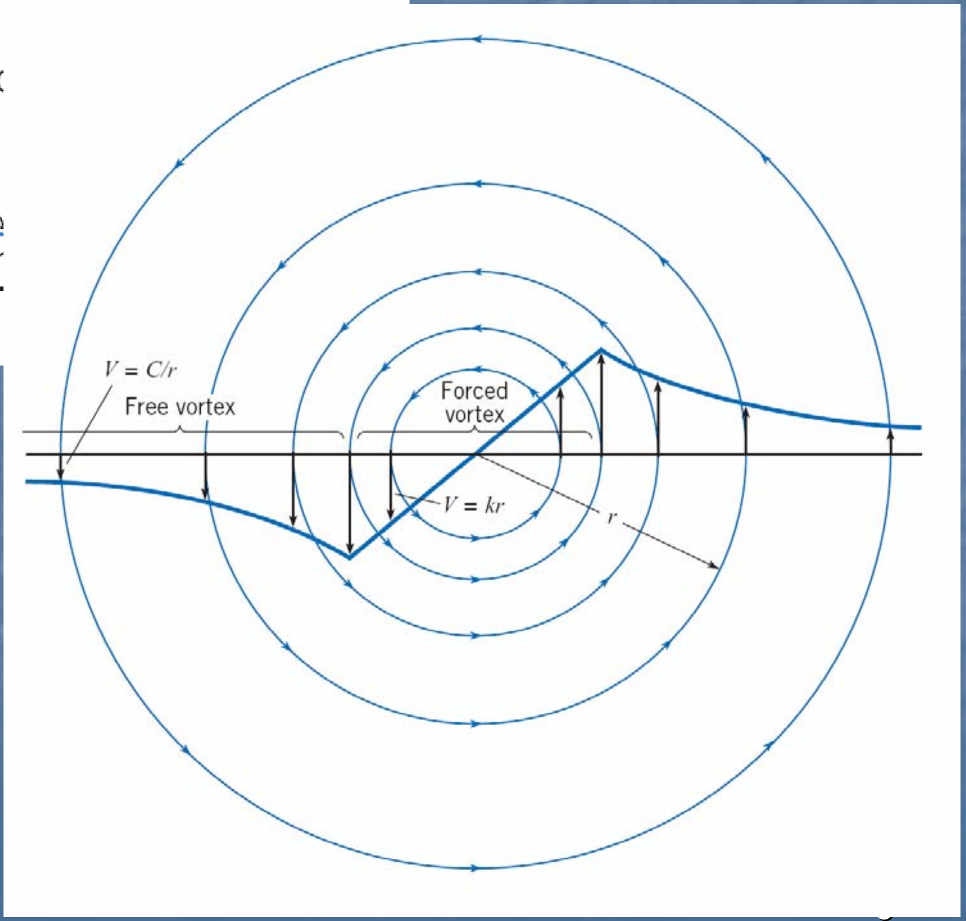
$$-\frac{d}{dr}(p + \gamma z) = -\rho V\left(\frac{dV}{dr}\right) = \frac{d}{dr}\left(\rho\frac{V^2}{2}\right)$$

$$\frac{d}{dr}\left(p + \gamma z + \rho\frac{V^2}{2}\right) = 0$$

$$\left(p + \gamma z + \rho\frac{V^2}{2}\right) = C$$

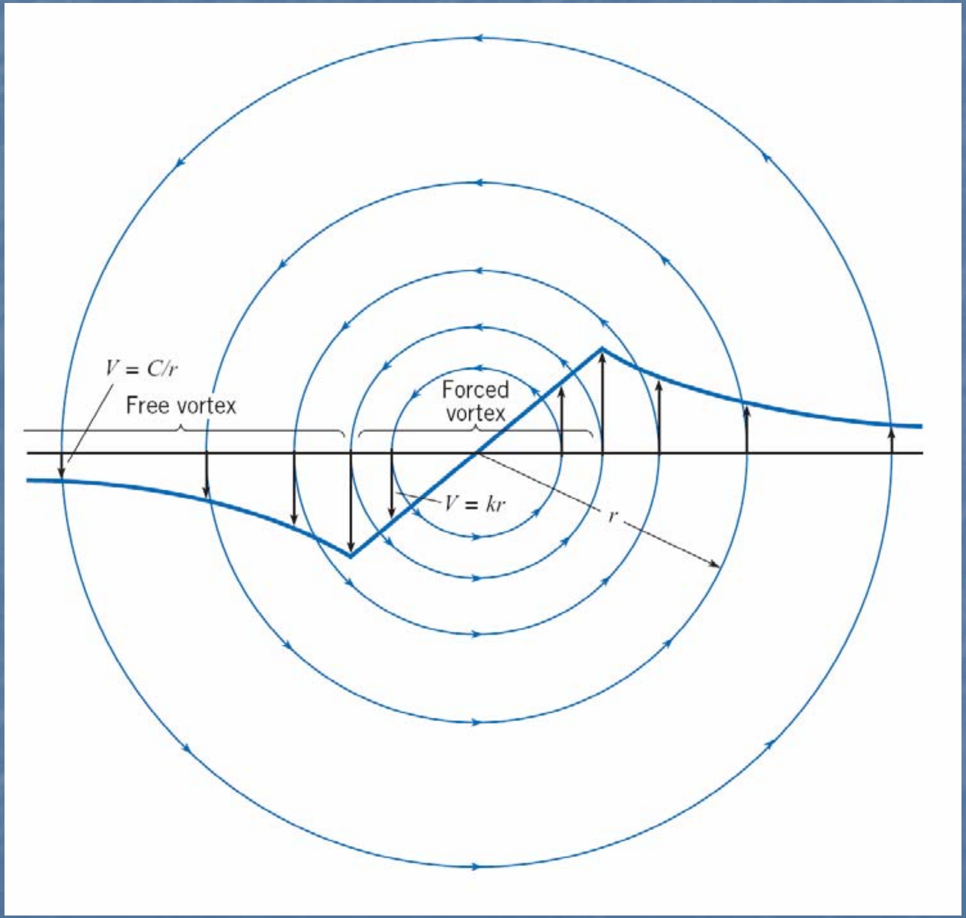
Pressure variation in a Tornado





Point 1 represent the forced vortex at the centre
Point 2 represent the junction between forced vortex and free vortex
Point 3 represent the free vortex where the velocity is zero and pressure is atmospheric.

Apply Bernoulli's equation to a free vortex between point 3 and any arbitrary point, we have

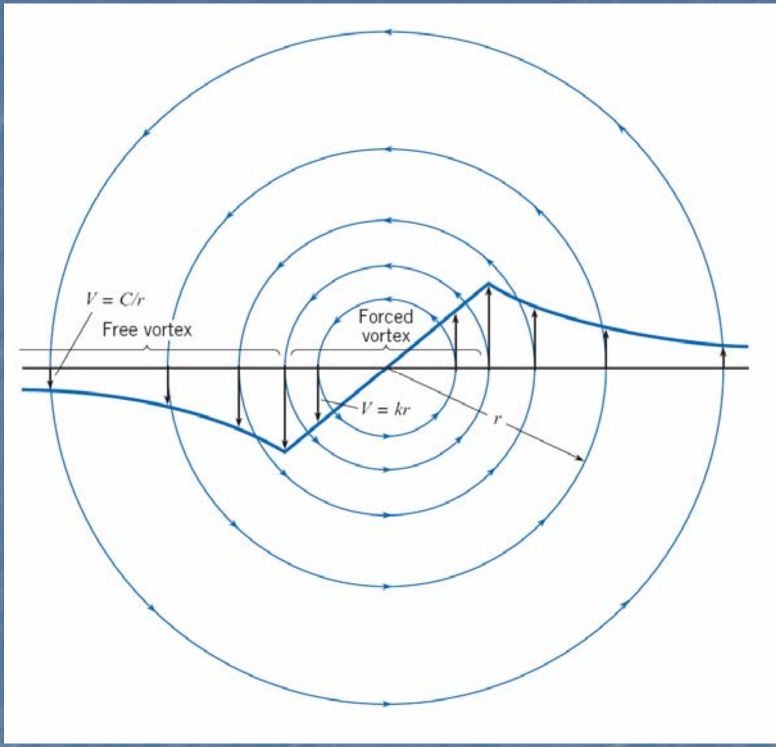


$$\left(p + \gamma z + \rho \frac{V^2}{2}\right) = \left(p_3 + \gamma z_3 + \rho \frac{V_3^{\ 2}}{2}\right)$$

$$V_3 = 0,\ p_3 = p_0,\ z = z_3$$

$$p - p_0 = -\rho \frac{V^2}{2}$$

Applying the equation for pressure variation in rotating flow between point 2 and point 3, we have



$$\left(p_2 + \gamma z_2 + \rho\frac{(V_2)^2}{2}\right) = \left(p_3 + \gamma z_3 + \rho\frac{V_3^{\,2}}{2}\right)$$

$$V_3 = 0,\ p_3 = p_0,\ z_2 = z_3$$

$$p_2 - p_0 = -\rho\frac{(V^2)_{max}}{2}$$

$$\left(p + \gamma z + \rho\frac{(\omega r)^2}{2}\right) = \left(p_2 + \gamma z_2 + \rho\frac{(\omega r_2)^2}{2}\right)$$

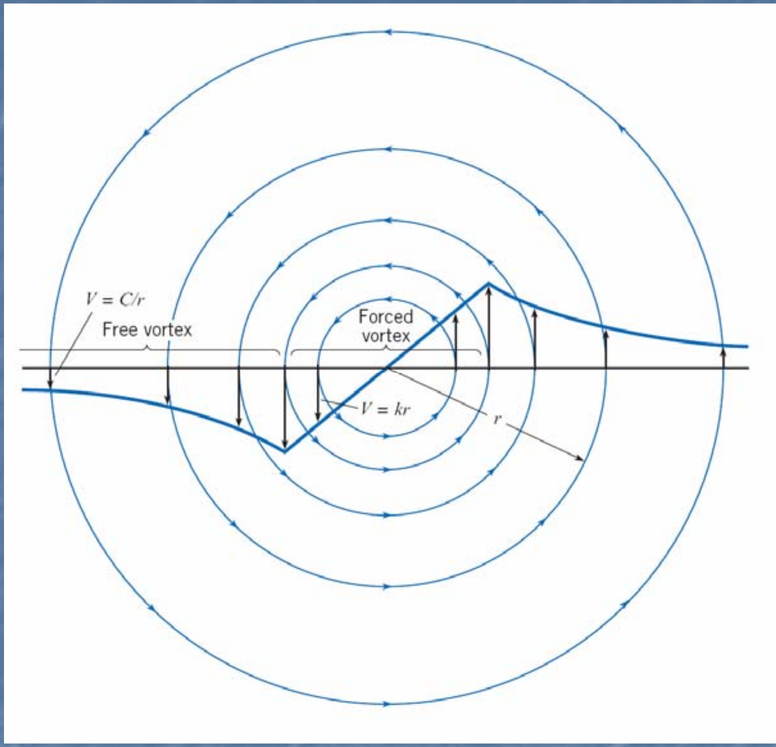
$$V_{max} = \omega r_2 \qquad z = z_2$$

$$p = p_2 - \rho\frac{(V)^2_{max}}{2} + \rho\frac{V^2}{2} \qquad p_2 - p_0 = -\rho\frac{(V^2)_{max}}{2} \qquad p = p_0 - \rho V^2_{max} + \rho\frac{V^2}{2}$$

The pressure difference between point (1) i.e. the center of the tornado where $(V = 0)$ and point (3) where $(p = p_0)$

$$p_1 - p_0 = -\rho V^2_{max}$$

Applying the equation for pressure variation in rotating flow between point (A) and point (B), we have



$$\left(p_A + \gamma z_A + \rho \frac{(V_A)^2}{2}\right) = \left(p_B + \gamma z_B + \rho \frac{V_B^{\,2}}{2}\right)$$

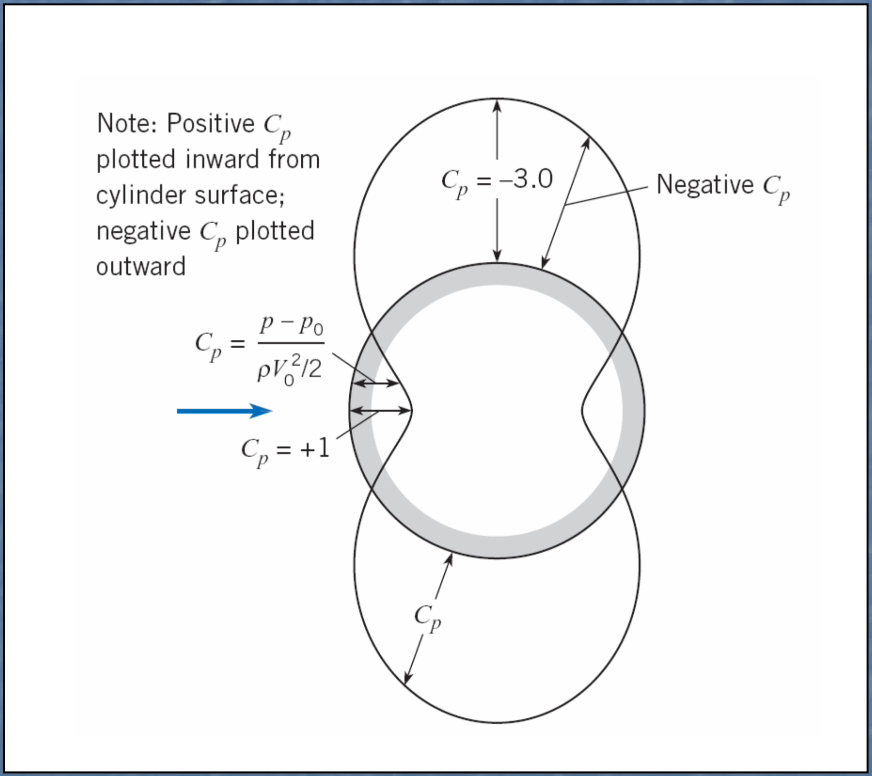
The pressure difference between point (1) i.e. the center of the tornado and the outer edge of the tornado

$$p_1 - p_0 = -\rho V_{\max}^2$$

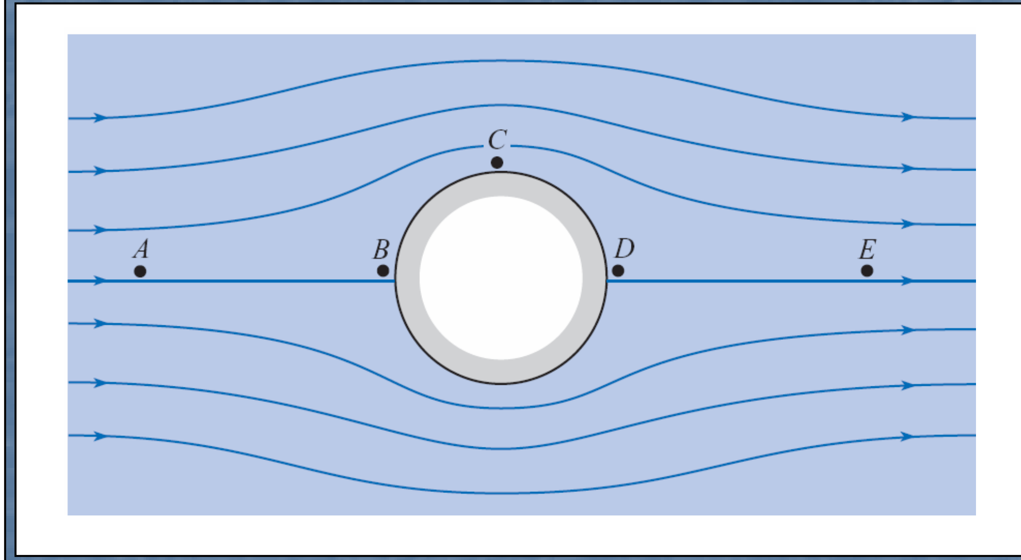
$V_{\max}^2$ **Is the given velocity at the junction between forced vortex and free vortex**

Pressure variation Around a Circular Cylinder

For an ideal fluid **(nonviscous , incompressible irrotational and steady),** the flow pattern as shown below.

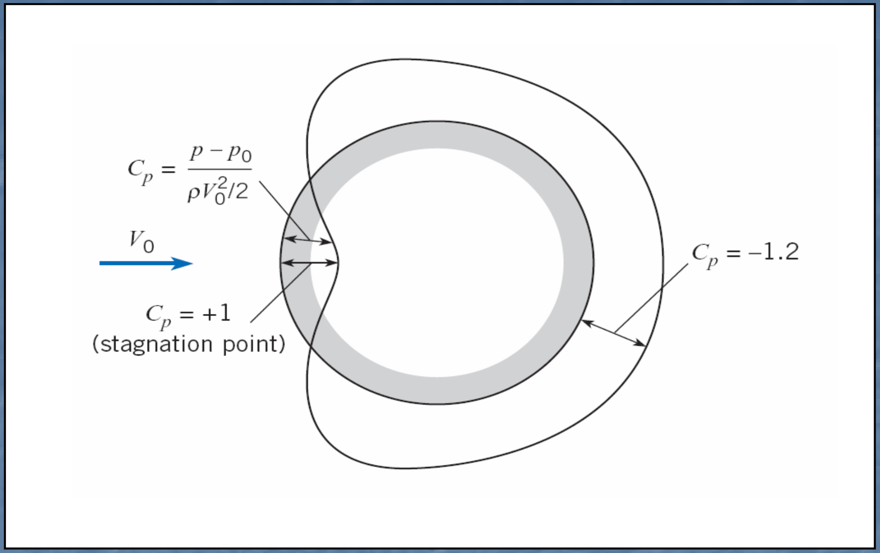


Pressure distribution on a cylinder irrotational flow

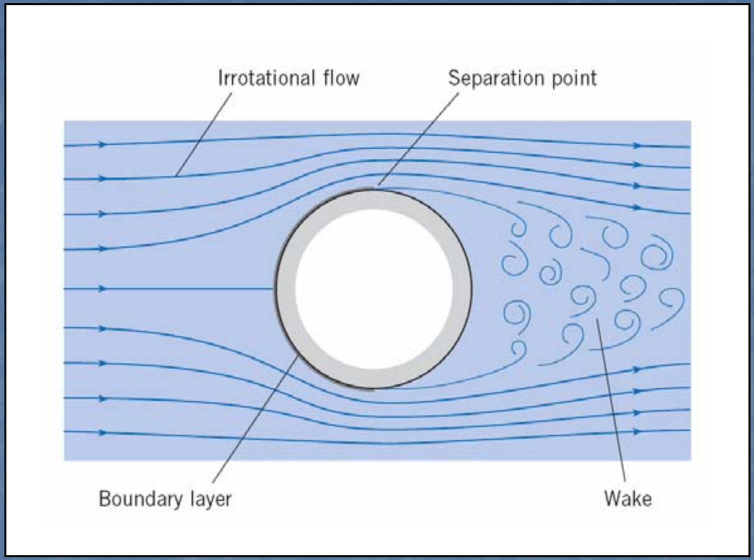


Irrotational flow past a cylinder

Separation



Pressure distribution on a circular cylinder *Re* = 105

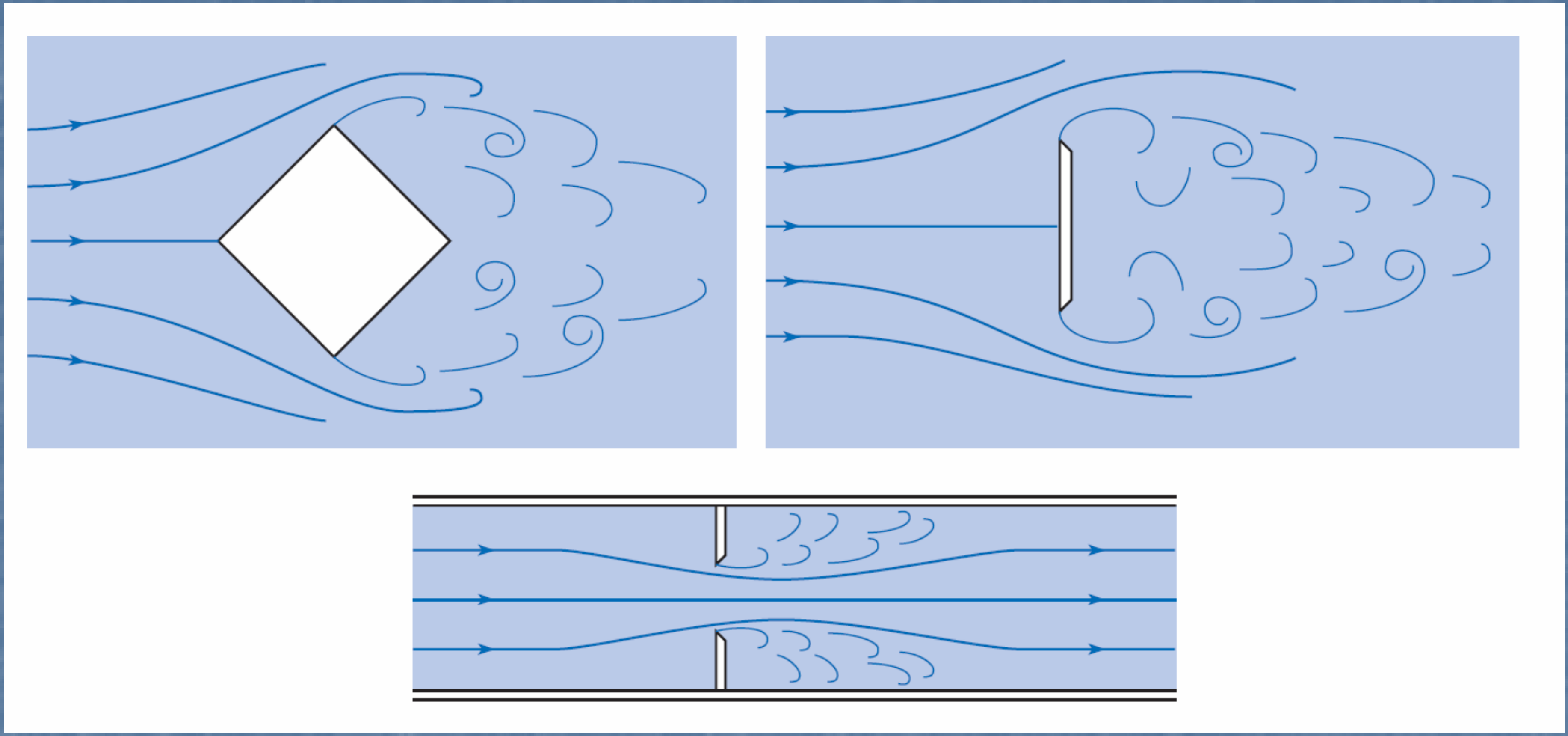


Flow of a real fluid past a circular cylinder

Separation point depends on:

(1) Free stream velocity; (2) Cylinder diameter; (3) Reynolds number; (4) Cylinder roughness; (5) Turbulence in the free stream.

Separation



Flow past a square rod and a disk and through a sharp-edged orifice

END OF LECTURE (7)